

## A Sampling Technique for Calibrating Phase Angle Generators from 1 Hz to 100 kHz

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### Summary Abstract

A method of calibrating phase angle generators from 1 Hz to 100 kHz is described. A commercial dual-channel waveform sampler is used to digitize both waveforms of the generator. The phase relationship between the two signals is resolved to  $0.001^\circ$  ( $17 \mu\text{rad}$ ) using a four parameter sine fit. The uncertainty in phase linearity is  $\pm 0.001^\circ$  to  $0.01^\circ$  over the above frequency range.

### Summary

Programmable phase angle generators for calibrating phase meters have been described in the literature [1,2] and are commercially available from several manufacturers. The calibration of these generators is normally performed at cardinal points ( $0^\circ$ ,  $90^\circ$ ,  $180^\circ$ , and  $270^\circ$ ) using bridge techniques [3]. The linearity between these cardinal points is tested by offsetting both channels by fixed phase angles and remeasuring the cardinal point. While this approach is useful, it has weaknesses i.e., the two signals are always either  $0^\circ$ ,  $90^\circ$  or  $180^\circ$  apart. In addition, it is difficult to automate.

An alternate technique for measuring phase linearity is based on waveform sampling. Using a commercially available, dual-channel, 16-bit, 1 million samples per second (MSPS) sampling system, both waveforms are digitized over  $n$  periods of the sinusoidal signal from the phase

generator. In order to overcome phase resolution problems due to sampler timebase limitations, some sort of signal processing must be performed on the waveform data. Performing a complex FFT will provide magnitude as well as phase information for each waveform. However, since the signals are nearly sinusoidal, only the phase relationship between the fundamental components in each waveform is of interest (commonly used bridge techniques measure only this component). Therefore, if the frequency is accurately known, a least squares sine fit is a more computationally efficient approach than an FFT. A three parameter fit provides a good figure for dc offset, amplitude, and phase.

In the most accurate phase angle generators, the output signals are digitally synthesized. The phase between these signals is shifted by modifying one of the sine lookup tables used in the synthesis process. The frequency of signals generated by this technique is as stable as the time base (typically  $<1$  ppm/h). However, the frequency uncertainty may be  $\pm 10$  ppm depending on the frequency synthesizer used. Therefore, a four parameter sine fit (including frequency) will often give better results.

The following algorithm has been employed to achieve phase resolution of  $<0.001^\circ$  in the audio frequency range:

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- Both signals are simultaneously sampled over a large integer ( $n > 500$ ) number of periods.
- A three parameter sine fit routine [4] is used on each waveform record to come up with reasonable estimates for amplitude, phase, and dc offset. Assuming a data record  $y_n$  of  $M$  samples, the solution of this fit is of the form

$$y_n = A\cos(\omega t_n) + B\sin(\omega t_n) + C$$

where

$\omega$  = the known angular input frequency.

$t_n$  = the time of each sample.

Only the frequency must be known for the algorithm to converge.

- The results from 2 are then used as initial estimates in a four parameter sine fit algorithm [4] whose solution is of the form

$$y = A\cos(\omega t + \theta) + C$$

The sum of squares of the error between this estimate and the measured data is given by

$$\epsilon = \sum [y_n - A\cos(\omega t_n + \theta) - C]^2$$

The routine iteratively chooses  $A$ ,  $\omega$ ,  $\theta$ , and  $C$  to minimize  $\epsilon$ . Because this is an iterative process, these parameters initially must be chosen close enough for the process to converge in a reasonable amount of time.

- The two input signals to the sampling instrument are reversed, and steps 1-3 are performed on this second set of data. This technique effectively cancels any differential

time delay present between the sampler's input channels.

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### References

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